EXAMPLE 11 Write a polynomial function $f(x)$ whose zeros are -1, 0 and 3.

SOLUTION Each zero of the polynomial function f yields a linear factor of the polynomial.

For $x = -1$, $x + 1 = 0$. **Writing a linear factor for each zero**
For $x = 0$, $x = 0$.
For $x = 3$, $x - 3 = 0$.

Thus a polynomial function with zeros -1, 0, and 3 is

$$f(x) = (x + 1) \cdot x \cdot (x - 3);$$

multiplying gives us

$$f(x) = x^3 - 2x^2 - 3x.$$

Try Exercise 71.

5.4 Exercise Set

FOR EXTRA HELP MyMathLab MathXL PRACTICE WATCH DOWNLOAD READ REVIEW

Concept Reinforcement *Classify each of the following statements as either true or false.*

1. When factoring any polynomial, it is always best to look first for a common factor. True
2. Whenever the sum of a negative number and a positive number is negative, the negative number has the greater absolute value. True
3. Whenever the product of a pair of factors is negative, the factors have the same sign. False
4. If $p + q = -17$, then $-p + (-q) = 17$. True
5. To factor $x^2 + 16x + 60$, consider only pairs of positive factors of 60. True
6. To factor $x^2 - 4x - 60$, consider only pairs of negative factors of 60. False
7. If 1 is a zero of a polynomial function, then $x - 1$ is a factor of the polynomial. True
8. If $x - 2$ is a factor of $p(x)$, then $p(2) = 0$. True

Factor completely. Remember to look first for a common factor. If a polynomial is prime, state this.

9. $x^2 + 8x + 12$ ⊡
10. $x^2 + 6x + 5$ ⊡
11. $t^2 + 8t + 15$ ⊡
12. $y^2 + 12y + 27$ ⊡
13. $a^2 - 7a + 12$ ⊡
14. $z^2 - 8z + 7$ ⊡
15. $x^2 - 2x - 15$ ⊡
16. $x^2 - x - 42$ ⊡
17. $x^2 + 2x - 15$ ⊡
18. $x^2 + x - 42$ ⊡
19. $2n^2 - 20n + 50$ ⊡
20. $2a^2 - 16a + 32$ ⊡
21. $a^3 - a^2 - 72a$ ⊡
22. $x^3 + 3x^2 - 54x$ ⊡
23. $14x + x^2 + 45$ ⊡
24. $12y + y^2 + 32$ ⊡
25. $3x + x^2 - 10$ ⊡
26. $x + x^2 - 6$ ⊡
27. $3x^2 - 15x + 18$ ⊡
28. $5y^2 - 40y + 35$ ⊡
29. $56 + x - x^2$ ⊡
30. $32 + 4y - y^2$ ⊡
31. $32y + 4y^2 - y^3$ ⊡
32. $56x + x^2 - x^3$ ⊡
33. $x^4 + 11x^3 - 80x^2$ ⊡
34. $y^4 + 5y^3 - 84y^2$ ⊡
35. $x^2 + 12x + 13$ ⊡
36. $x^2 - 3x + 7$ ⊡
37. $p^2 - 5pq - 24q^2$ ⊡
38. $x^2 + 12xy + 27y^2$ ⊡
39. $y^2 + 8yz + 16z^2$ ⊡
40. $x^2 - 14xy + 49y^2$ ⊡
41. $p^4 - 80p^3 + 79p^2$ ⊡
42. $x^4 - 50x^3 + 49x^2$ ⊡

43. Use the results of Exercise 9 to solve $x^2 + 8x + 12 = 0$. $-6, -2$

44. Use the results of Exercise 10 to solve $x^2 + 6x + 5 = 0$. $-5, -1$

45. Use the results of Exercise 19 to solve $2n^2 + 50 = 20n$. 5

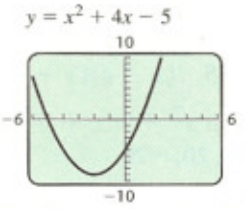
46. Use the results of Exercise 30 to solve $32 + 4y = y^2$. $-4, 8$

⊡ Answers to Exercises 9–42 are on p. IA-12.

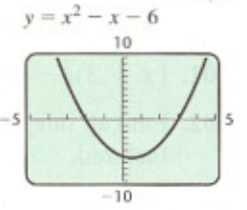
In Exercises 47–50, use the graph to solve the given equation. Check by substituting into the equation.

Aha!

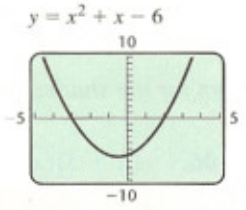
47. $x^2 + 4x - 5 = 0$ $-5, 1$



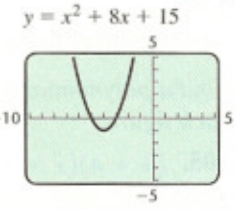
48. $x^2 - x - 6 = 0$ $-2, 3$



49. $x^2 + x - 6 = 0$ $-3, 2$



50. $x^2 + 8x + 15 = 0$ $-5, -3$



Find the zeros of each function.

51. $f(x) = x^2 - 4x - 45$ $-5, 9$

52. $f(x) = x^2 + x - 20$ $-5, 4$

53. $r(x) = x^3 + 4x^2 + 3x$ $-3, -1, 0$

54. $g(x) = 3x^2 - 21x + 30$ $2, 5$

Solve.

55. $x^2 + 4x = 45$ $-9, 5$

56. $t^2 - 3t = 28$ $-4, 7$

57. $x^2 - 9x = 0$ $0, 9$

58. $a^2 + 18a = 0$ $-18, 0$

59. $a^3 + 40a = 13a^2$ $0, 5, 8$

60. $x^3 - 2x^2 = 63x$ $-7, 0, 9$

61. $(x - 3)(x + 2) = 14$ $-4, 5$

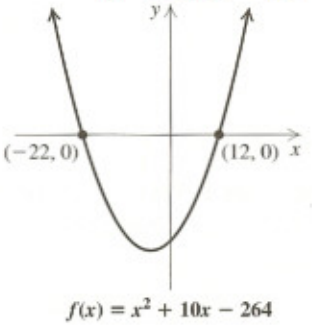
62. $(z + 4)(z - 2) = -5$ $-3, 1$

63. $35 - x^2 = 2x$ $-7, 5$

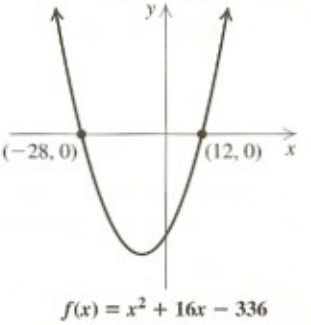
64. $40 - x^2 + 3x = 0$ $-5, 8$

In Exercises 65 and 66, use the graph to factor the given polynomial.

65. $x^2 + 10x - 264$
$(x + 22)(x - 12)$



66. $x^2 + 16x - 336$
$(x + 28)(x - 12)$



In Exercises 67–70, use a graph to help factor each polynomial.

67. $x^2 + 40x + 384$
$(x + 24)(x + 16)$

68. $x^2 - 13x - 300$
$(x + 12)(x - 25)$

69. $x^2 + 26x - 2432$
$(x + 64)(x - 38)$

70. $x^2 - 46x + 504$
$(x - 18)(x - 28)$

Answers to Exercises 71–76 and 87 are on p. IA-12.

Write a polynomial function that has the given zeros. Answers may vary.

71. $-1, 2$

72. $2, 5$

73. $-7, -10$

74. $8, -3$

75. $0, 1, 2$

76. $-3, 0, 5$

TW **77.** Allison says that she will never miss a point of intersection when solving graphically because she always uses a $[-100, 100, -100, 100]$ window. Is she correct? Why or why not?

TW **78.** Shari factors $x^3 - 8x^2 + 15x$ as $(x^2 - 5x)(x - 3)$. Is she wrong? Why or why not? What advice would you offer?

SKILL REVIEW

To prepare for Section 5.5, review multiplying binomials using FOIL (Section 5.2).

Multiply. [5.2]

79. $(2x + 3)(3x + 4)$
$6x^2 + 17x + 12$

80. $(2x + 3)(3x - 4)$
$6x^2 + x - 12$

81. $(2x - 3)(3x + 4)$
$6x^2 - x - 12$

82. $(2x - 3)(3x - 4)$
$6x^2 - 17x + 12$

83. $(5x - 1)(x - 7)$
$5x^2 - 36x + 7$

84. $(x + 6)(3x - 5)$
$3x^2 + 13x - 30$

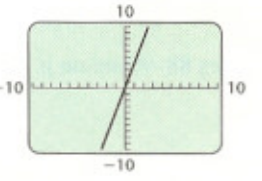
SYNTHESIS

TW **85.** Explain how the following graph of

$$y = x^2 + 3x - 2 - (x - 2)(x + 1)$$

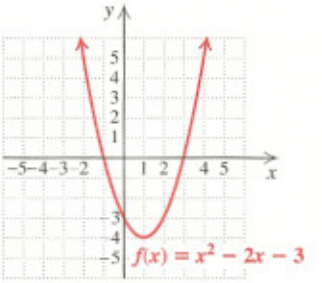
can be used to show that

$$x^2 + 3x - 2 \neq (x - 2)(x + 1).$$

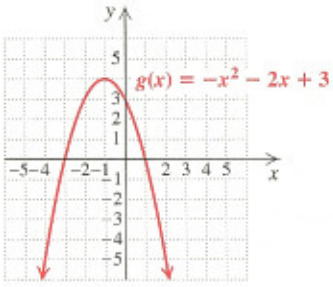


TW **86.** When searching for a factorization, why do we list pairs of numbers with the correct *product* instead of pairs of numbers with the correct *sum*?

87. Use the following graph of $f(x) = x^2 - 2x - 3$ to solve $x^2 - 2x - 3 = 0$ and to solve $x^2 - 2x - 3 < 5$.



88. Use the following graph of $g(x) = -x^2 - 2x + 3$ to solve $-x^2 - 2x + 3 = 0$ and to solve $-x^2 - 2x + 3 \geq -5$. ⊡



89. Find a polynomial function f for which $f(2) = 0$, $f(-1) = 0$, $f(3) = 0$, and $f(0) = 30$. ⊡

90. Find a polynomial function g for which $g(-3) = 0$, $g(1) = 0$, $g(5) = 0$, and $g(0) = 45$. ⊡

In Exercises 91–94, use a graphing calculator to find any solutions that exist accurate to two decimal places.

91. $-x^2 + 13.80x = 47.61$ 6.90

92. $-x^2 + 3.63x + 34.34 = x^2$ −3.33, 5.15

93. $x^3 - 3.48x^2 + x = 3.48$ 3.48

94. $x^2 + 4.68 = 1.2x$ No real solution

Factor. Assume that variables in exponents represent positive integers.

95. $x^2 + \frac{1}{2}x - \frac{3}{16}$ $\left(x + \frac{3}{4}\right)\left(x - \frac{1}{4}\right)$

96. $y^2 + 0.4y - 0.05$ $(y - 0.1)(y + 0.5)$

97. $x^{2a} + 5x^a - 24$ $(x^a + 8)(x^a - 3)$

⊡ Answers to Exercises 88–90 are on p. IA-12.

98. $a^2p^{2a} + a^2p^a - 2a^2$ $a^2(p^a + 2)(p^a - 1)$

Aha! **99.** $(a + 1)x^2 + (a + 1)3x + (a + 1)2$ $(a + 1)(x + 2)(x + 1)$

100. $ax^2 - 5x^2 + 8ax - 40x - (a - 5)9$ (*Hint*: See Exercise 99.) $(a - 5)(x + 9)(x - 1)$

Aha! **101.** $(x + 3)^2 - 2(x + 3) - 35$ $(x - 4)(x + 8)$

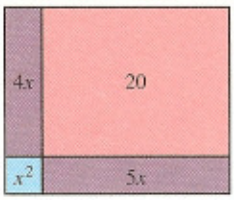
102. Find all integers m for which $x^2 + mx + 75$ can be factored. 76, −76, 28, −28, 20, −20

103. Find all integers q for which $x^2 + qx - 32$ can be factored. 31, −31, 14, −14, 4, −4

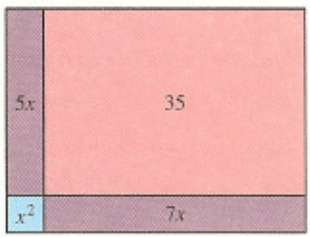
104. One factor of $x^2 - 345x - 7300$ is $x + 20$. Find the other factor. $x - 365$

Find a polynomial in factored form for the shaded area in each figure.

105. $(x + 4)(x + 5)$



106. $(x + 5)(x + 7)$



Try Exercise Answers: Section 5.4

9. $(x + 2)(x + 6)$ **13.** $(a - 3)(a - 4)$ **21.** $a(a + 8)(a - 9)$ **25.** $(x + 5)(x - 2)$ **35.** Prime **37.** $(p - 8q)(p + 3q)$ **43.** $-6, -2$ **59.** $0, 5, 8$ **61.** $-4, 5$ **67.** $(x + 24)(x + 16)$ **71.** $f(x) = (x + 1)(x - 2)$, or $f(x) = x^2 - x - 2$

5.5 Trinomials of the Type $ax^2 + bx + c$

- Factoring Trinomials of the Type $ax^2 + bx + c$
- Equations and Functions

FACTORING TRINOMIALS OF THE TYPE $ax^2 + bx + c$

Now we look at trinomials in which the leading coefficient is not 1. We consider two factoring methods. Use the method that you prefer or the one recommended by your instructor.

Method 1: Factoring with FOIL

We first consider the **FOIL method** for factoring trinomials of the type

$$ax^2 + bx + c, \quad \text{where } a \neq 1.$$